Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level In Mathematics (WMA11) Paper 01R

Question Number	Scheme	Marks
1.	$\int\left(\frac{1}{2}x^3+\frac{3}{\sqrt{x}}-4\right)\mathrm{d}x=\frac{1}{2}\times\frac{x^4}{4}+3\times\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-4x+c$	M1, A1
	$=\frac{1}{8}x^4+6x^{\frac{1}{2}}-4x+c$	A1, A1
		(4 marks)

M1: For raising any correct index by 1. So, award for $x^3\to x^4$, $x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$ or $-4\to-4x$ (allowing $-4x^1$)
The index must be processed and not left, for example, as x^{3+1}

A1: For **two** of $\frac{1}{2}\times\frac{x^4}{4}$, $+3\times\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $-4x$ (allowing $-4x^1$) correct simplified or correct unsimplified.

A1: For **two** of $\frac{1}{8}x^4$, $+6x^{\frac{1}{2}}$, $-4x$ correct and in simplest form.

Accept forms such as $0.125x^4$, $+6\sqrt{x}$ and $-4x^1$ and $+-4x^1$

A1: Fully correct, and simplified with $+c$ all on a single line. Accept simplified equivalents (see above)

Ignore any spurious notation such as $\int \frac{1}{8}x^4+6x^{\frac{1}{2}}-4x+c$ or $\frac{1}{8}x^4+6x^{\frac{1}{2}}-4x+c\ \mathrm{d}x$

If they go on, however, and multiply for example by 8 then it is A0 or integrate again it is A0.
They can do this in one line for full marks

Question Number	Scheme	Marks
2. (a)	$5(x+3)>4(2x-5)\Rightarrow 5x+15>8x-20\Rightarrow ax>b$ or $px<q$	M1
	$\Rightarrow x<\dfrac{35}{3}$	A1
		(2)
(b) (i)	$x^2-6x+1=(x-3)^2\pm...=(x-3)^2-8$	M1, A1
(ii)	$(x-3)^2-8=0\Rightarrow x=3+\sqrt{8}$ or $3-\sqrt{8}$	M1
	$x^2-6x+1\geqslant 0\Rightarrow x\leqslant 3-\sqrt{8}, x\geqslant 3+\sqrt{8}$	A1
		(4)
(c)	$x\leqslant 3-\sqrt{8},\ 3+\sqrt{8}\leqslant x<\dfrac{35}{3}$	B1
		(1) **(7 marks)**

This is effectively a non- calculator question and methods must be clearly shown.

(a)

M1: Attempts to expand brackets and then collect terms. Look for

- $5x\pm15>8x\pm20$
- followed by $ax>b$ or $px<q$

Also condone with an = sign in place of the inequality

A1: $x<\dfrac{35}{3}$ or exact equivalent such as $\dfrac{35}{3}>x$ **following the award of M1.**

Do not accept $x<11.66$ or $x<11.7$ but $x<11.\dot{6}$ is acceptable and ISW after a correct answer

Mark part (b) as one question

(b) (i)

M1: Starts the process by writing x^2-6x+1 as $(x-3)^2\pm c$, where c is a constant (which may not be simplified)

A1: $x^2-6x+1=(x-3)^2-8$. It is acceptable to write this down for the two marks

(b) (ii) **Note that this must follow on from their (b)(i) and CANNOT be restarted via the quadratic formula**

M1: Correctly solves their $(x+a)^2-b=0,\ b>0$ and finds at least one critical value.

Look for an intermediate line of working such as $(x+a)^2=b$ or $x+a=\pm\sqrt{b}$

Do not allow decimals, i.e. awrt 5.83, 0.17 but isw after a correct (exact) answer

It can be implied by an attempt to solve their inequality as long as an intermediate line is seen.

E.g. $(x+a)^2-b\geqslant 0,\ b>0\Rightarrow x+a...\sqrt{b}\Rightarrow x...-a+\sqrt{b}$ where … can even be an incorrect inequality

Again, it is important to see an intermediate line of working.

A1: $x\leqslant 3-\sqrt{8}, x\geqslant 3+\sqrt{8}$ o.e following the award of M1

ISW if this is subsequently changed to decimals. Accept $\sqrt{8}$ as $2\sqrt{2}$

(c)

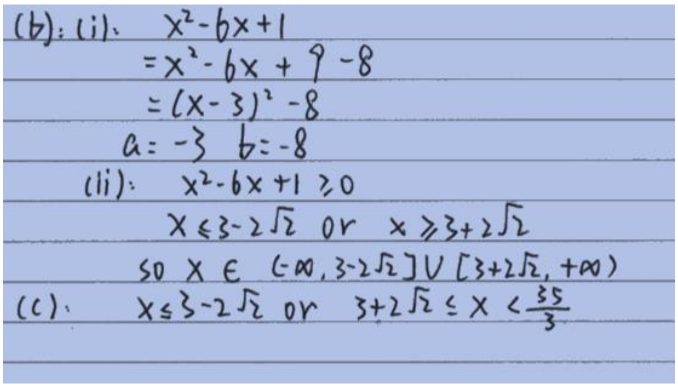
B1: CAO $x \leqslant 3-\sqrt{8},\ 3+\sqrt{8} \leqslant x < \frac{35}{3}$ o.e. It must be given as two separate inequalities.

Allow set notation $\left\{x \in \mathbb{R} : x \leqslant 3-\sqrt{8}\right\} \cup \left\{x \in \mathbb{R} : 3+\sqrt{8} \leqslant x < \frac{35}{3}\right\}$

A list $x < \frac{35}{3},\ x \leqslant 3-\sqrt{8},\quad x \geqslant 3+\sqrt{8}$ is B0.

There must be a correct attempt to state the two separate regions.

Condone a solution with 'and' between. E.g. $x \leqslant 3-\sqrt{8}$ and $3+\sqrt{8} \leqslant x < \frac{35}{3}$



(b): (i): x^2-6x+1

$=x^2-6x+9-8$

$=(x-3)^2-8$

$a=-3 \quad b=-8$

(ii): $x^2-6x+1 \geqslant 0$

$x \leqslant 3-2\sqrt{2}$ or $x \geqslant 3+2\sqrt{2}$

so $x \in (-\infty, 3-2\sqrt{2}] \cup [3+2\sqrt{2}, +\infty)$

(c): $x \leq 3-2\sqrt{2}$ or $3+2\sqrt{2} \leq x < \frac{35}{3}$

Example of how to mark:

(b)(i) M1, A1: Completely correct

(ii) M0, A0: No intermediate line. This could have been done on a calculator

(c) B1: Completely correct answer

Question Number	Scheme	Marks
3.(a)	$\left(\frac{dy}{dx}\right) = 3x^2 + \frac{48}{\sqrt{x}}$	M1, A1, A1
		(3)
(b)	$\left(\frac{d^2y}{dx^2}\right) = 6x - \frac{24}{x^{\frac{3}{2}}}$	M1
	$\Rightarrow x^{\frac{5}{2}} = ...$ $\quad\quad x = 2^{\frac{4}{5}}$	dM1 A1
		(3) **(6 marks)**

(a)

M1: For reducing a correct power by one any term. So, award for $x^3 \to x^2$, $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or even $5 \to 0$
As in question 1, the indices must be processed

A1: One of the two terms, correct **and** simplified, either $3x^2$ or $+\frac{48}{\sqrt{x}}$ or exact simplified equivalent

A1: CAO $3x^2 + \frac{48}{\sqrt{x}}$ or exact simplified equivalent such as $3x^2 + 48x^{-\frac{1}{2}}$

(b)

M1: For reaching $\left(\frac{d^2y}{dx^2}\right) = Ax \pm \frac{B}{x^{\frac{3}{2}}}$ o.e where A and B are non-zero constants

dM1: Sets their $\frac{d^2y}{dx^2} = Ax \pm \frac{B}{x^{\frac{3}{2}}} = 0$ and proceeds to $x^{\frac{5}{2}} = C$ (or equivalent working) where C is a constant.

Example of equivalent working $x = \frac{4}{x^{\frac{3}{2}}} \Rightarrow x^2 = \frac{16}{x^3} \Rightarrow x^5 = 16$

A1: CAO $x = 2^{\frac{4}{5}}$ following correct $\frac{d^2y}{dx^2}$ with dM1 also being scored. If it is achieved via decimals it is A0

Condone sight of correct work with correct k. E.g. $x^{\frac{5}{2}} = 4 \Rightarrow k = \frac{4}{5}$

Question Number	Scheme	Marks
4.(a)	Sets $kx+2=\frac{2}{x}+3x-4$ and attempts to collect terms or multiply through by x	M1
	$(k-3)x+6-\frac{2}{x}=0 \Rightarrow (k-3)x^2+6x-2=0$ *	A1*
		(2)
(b)	Attempts b^2-4ac for $(k-3)x^2+6x-2=0$	M1
	Solves $b^2-4ac=0$ for $(k-3)x^2+6x-2=0 \Rightarrow 36+8(k-3)=0 \Rightarrow k=...$	dM1
	$k=-\frac{3}{2}$	A1
		(3)
		(5 marks)

Ignore labels here. Mark (a) and (b) as one complete question.

(a)

M1: Sets $kx+2=\frac{2}{x}+3x-4$ condoning slips and either attempts to collect terms or else multiply through by x

Allow for either of

- $kx+2=\frac{2}{x}+3x-4 \Rightarrow (k\pm 3)x+6=\frac{2}{x}$ o.e condoning slips on the 6. E.g. condone $6 \leftrightarrow -2$
- $kx+2=\frac{2}{x}+3x-4 \Rightarrow kx^2+2x=2+3x^2-4x$ with correct multiplication seen on at least 3 terms

A1*: Proceeds to given answer showing at least the required/ necessary steps with no incorrect lines. Examples of correct work gaining M1 A1 showing a minimum number of necessary steps (three)

- By initially collecting terms: $kx+2=\frac{2}{x}+3x-4 \Rightarrow (k-3)x+6-\frac{2}{x}=0 \Rightarrow (k-3)x^2+6x-2=0$
- By multiplying initially: $kx+2=\frac{2}{x}+3x-4 \Rightarrow kx^2+2x=2+3x^2-4x \Rightarrow (k-3)x^2+6x-2=0$
- Bracketing must be correct. There may be extra lines, which is fine, as long as they are correct.

(b)

M1: Attempts b^2-4ac for $(k-3)x^2+6x-2=0$ with **correct** values for a and b with c condoned as ± 2

This is implied by setting $b^2=4ac$ but also $b^2-4ac>0$ and $b^2-4ac<0$ as well as non-strict versions. Note that stating and attempting b^2+4ac is M0 followed by dM0, A0

This cannot be scored if b^2-4ac can only be seen embedded within a quadratic formula.

dM1: Solves $b^2-4ac=0$ o.e. for $(k-3)x^2+6x-2=0 \Rightarrow 6^2-4(k-3)(-2)=0 \Rightarrow k=....$

Also $(k-3)x^2+6x-2=0 \Rightarrow 36+8(k-3)=0 \Rightarrow k=...$

It is dependent upon the previous M1 being an **equality** AND **correct** a, b and c leading to a value for k

Don't be too concerned by the processing of the equation leading to the value for k

A1: $k=-\frac{3}{2}$ o.e.

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Alt (b) via gradients.

M1: Sets $\frac{dy}{dx} = \frac{-2}{x^2} + 3 = k$. Look for a $\frac{dy}{dx}$ in the form $\frac{\alpha}{x^2} + \beta$

dM1: Attempts to find $x^2 = \left(\frac{2}{3-k}\right)$, substitutes into $(k-3)x^2 + 6x - 2 = 0$ and solves to find $k = ...$

Alternatively substitutes $k = \frac{-2}{x^2} + 3$ into $(k-3)x^2 + 6x - 2 = 0$ and solves to find x and then k.

A1: $k = -\frac{3}{2}$ o.e such as $-\frac{12}{8}$ or -1.5

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Question Number	Scheme	Marks
5 (a)	$9x^3-10x^2+x=x\left(9x^2-10x+1\right)=x(ax\pm1)(bx\pm1)$ with $ab=9$	M1
	$=x(9x-1)(x-1)$	A1
		(2)
(b)	States or implies that $x=3^y$ AND sets = to their 1 or $\frac{1}{9}$	M1
	Solves their $3^y="1"$ or $3^y="\frac{1}{9}"$	dM1
	$y=0,-2$	A1
		(3)
		(5 marks)

THIS IS A NON- CALCULATOR QUESTION.

(a)

M1: Takes out (or divides by) a common factor of x and attempts to factorise the quadratic factor. In most cases this will be

- either $9x^3-10x^2+x=x\left(9x^2-10x+1\right)=x(ax\pm1)(bx\pm1)$ with $ab=9$
- or $9x^2-10x+1=(ax\pm1)(bx\pm1)$ with $ab=9$

Other methods exist such as $9x^3-10x^2+x=(x-1)\left(9x^2-x\right)$ but the method is only scored when the factor of x is taken from the second bracket

It would be very difficult to score following an attempt to find the roots of $9x^2-10x+1=0$

A1: $x(9x-1)(x-1)$ o.e such as $(9x-1)(x-1)(x-0)$, $x(-9x+1)(-x+1)$ or $9x\left(x-\frac{1}{9}\right)(x-1)$ following the award of the M1 mark. **This must be done in part (a) but use review if unsure**

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Watch out for these.

Example 1: If you see $9x^3-10x^2+x=0\Rightarrow x=0,1,\frac{1}{9}$ followed by $x(9x-1)(x-1)$ it is M0 A0 (calculator use)

Example 2: If you see $9x^3-10x^2+x=x\left(x-\frac{1}{9}\right)(x-1)$ it is M0 A0 (Incorrect and probable calculator use)

Special case:

If you see $9x^3-10x^2+x=x(9x-1)(x-1)$ or $9x\left(x-\frac{1}{9}\right)(x-1)$ without any other working it is SC M1 A0

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(b) If a candidate has just written down the roots of part (a) =0 without having factorised it, then allow all marks to be scored here. So, 00 111 is possible.

M1: This is awarded for linking the equation in *y* with the roots of the part (a)

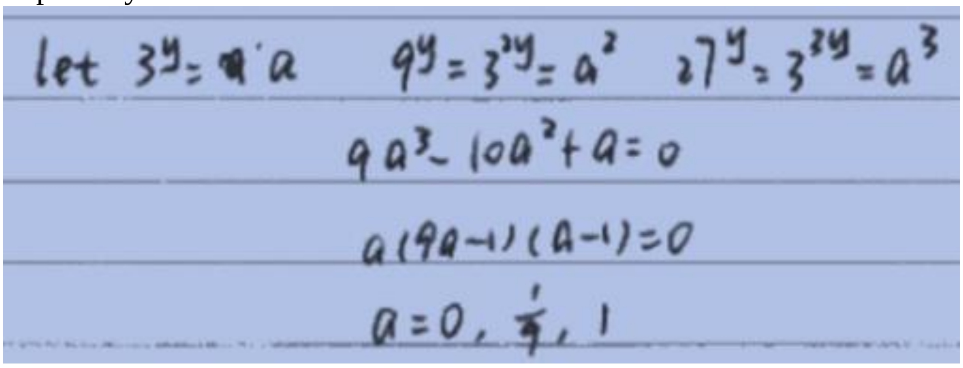
Score for stating that $x = 3^y$ **AND** setting *x* or 3^y = one of the non-zero roots of their $x(9x-1)(x-1) = 0$

It is important to note that it is not just for the non-zero roots of their $x(9x-1)(x-1) = 0$

This can be implied by setting $3^y = 1$ OR $\frac{1}{9}$ if they write down the roots of part (a) as 0, 1 and $\frac{1}{9}$

Can be implied by $x = 1$ OR $\frac{1}{9}$ followed by a correct *y* value.

It is also implied by this sort of work as $a = 3^y$



dM1: Solves their $3^y = "1"$ or $3^y = "\frac{1}{9}"$.

To score this, the root must be either correct or else a power of 3

A1: CSO $y = 0, -2$ only.

The solutions must follow M1, dM1. Candidates cannot just state $y = 0$ without any working and score marks.

Condone use of ‘or’ between the values.

Condone the use of logs in proceeding to an answer. E.g. $3^y = \frac{1}{9} \Rightarrow y = \log_3 \frac{1}{9} = -2$

Question Number	Scheme	Marks
6 (a)	Angle $AFB = \frac{\pi - 2.275}{2} = 0.433$ *	B1*
		(1)
(b)	Attempts $r\theta = 6.2 \times 2.275 = (14.105)$	M1
	Attempts $(x^2) = 6.4^2 + 6.2^2 - 2\times 6.4 \times 6.2\cos(0.433) = (7.36)$ or $(x = 2.71)$	M1
	Correct attempt = $2x + r\theta + 12.8 = \ 2\times 2.714 + 14.105 + 12.8 = 32.3(\text{m})$	dM1, A1
		(4)
(c)	Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6.2^2 \times 2.275 = (43.7255)$	M1
	Attempts $\frac{1}{2}ab\sin C = \frac{1}{2}\times 6.2\times 6.4\times \sin 0.433 = (8.325)$	M1
	Correct attempt = $2\times 8.325 + 43.7255 = 60.4\left(\text{m}^2\right)$	dM1, A1
		(4)
		(9 marks)

(a)

B1*: Shows that angle $AFB = 0.433$ radians.

You may see the angle labelled θ or BFA. This is not an issue, just look for the calculation

It is a "show that" so it is important to see either $\frac{\pi - 2.275}{2}$ or decimal equivalent with at least this accuracy $\frac{3.1416 - 2.275}{2}$ before we see 0.433 written down. The bracketing if required must be correct. So $(\pi - 2.275)\div 2$ is correct but $\pi - 2.275 \div 2$ is not. Condone a more accurate answer being written down and left without rounding to 3 decimal places. E.g. $\frac{\pi - 2.275}{2} = 0.433296327$

(b)

M1: Attempts $r\theta = 6.2\times 2.275$. Implied by sight of awrt 14.1

Sight of $2\times 6.2\times 2.275$ would be M0 as this would imply an incorrect formula. There are not two arc lengths within the perimeter of the tunnel.

M1: Attempts to apply the cosine rule to find *AB* or *DE*.

Can be achieved for an attempt at $6.4^2 + 6.2^2 - 2\times 6.4\times 6.2\cos(0.433)$ irrespective of the LHS.

Condone a larger calculation that includes $2\times\left(\sqrt{6.4^2 + 6.2^2 - 2\times 6.4\times 6.2\cos(0.433)}\right)$ or even

$2\times\left(6.4^2 + 6.2^2 - 2\times 6.4\times 6.2\cos(0.433)\right)$ as both *AB* and *DE* are part of the perimeter.

The angle used must be 0.433 or a more accurate version of this

This can be implied by, for example, by sight of $AB^2 = 7.36...$ OR $AB = 2.71...$

Condone $\cos 0.433 = \frac{6.2^2 + 6.4^2 - AB^2}{2\times 6.2\times 6.4}$ leading to a value for AB^2 or *AB*

Attempts via right angled triangles and incorrect cosine rules, e.g. $\left(c^2\right) = a^2 + b^2 - ab\cos C$ are M0

dM1: Full attempt to find the perimeter of the tunnel. $2\times"2.714"+"14.105"+12.8$

It is dependent upon both M's and a correct attempt at the cosine rule by square rooting their answer to $6.4^2 + 6.2^2 - 2\times 6.4\times 6.2\cos(0.433)$ o.e.

A1: awrt $32.3(\text{m})$

(c)

M1: Attempts $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6.2^2 \times 2.275$. Implied by awrt 43.7

M1: Attempts either $\frac{1}{2}ab\sin C = \frac{1}{2}\times 6.2\times 6.4\times \sin 0.433$ or $ab\sin C = 6.2\times 6.4\times \sin 0.433$
The angle used must be 0.433 or a more accurate version of this.

dM1: Full attempt to find the cross-sectional area of the tunnel.
It is dependent upon both M's and an attempt to add a sector and two triangles

A1: awrt $60.4\left(\mathrm{m}^2\right)$

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Answers using degrees can still be awarded ALL of the marks.

In part (a) it would be difficult to score the mark but you should expect to see the following calculations

$$\text{angle } BFD = 2.275\times\frac{180}{\pi} = \text{awrt } 130.3°$$

$$\text{angle } AFB = \frac{180° - 130.34°}{2} = \text{awrt } 24.8° = 24.8\times\frac{\pi}{180} = 0.433$$

For parts (b) and (c) the calculations must be attempted using angles with (at least) the following accuracy.

$$\text{angle } BFD = \text{ awrt } 130° \text{ and angle } AFB = \text{ awrt } 25°$$

For (b)

M1: Attempts $\frac{\theta}{360}\times 2\pi r = \frac{130}{360}\times 2\pi\times 6.2$

M1: Attempts $\left(x^2\right) = 6.4^2 + 6.2^2 - 2\times 6.4\times 6.2\cos 25°$

For (c)

M1: Attempts $\frac{\theta}{360}\times \pi r^2 = \frac{130}{360}\times\pi\times 6.2^2$

M1: Attempts $\frac{1}{2}ab\sin C = \frac{1}{2}\times 6.2\times 6.4\times \sin 25$ or $ab\sin C = 6.2\times 6.4\times\sin 25$

Question Number	Scheme	Marks
7 (a)	States or implies that B is $(10, 6)$	B1
	$(AB^2) = (10-0)^2 + ("6"-2)^2 \Rightarrow AB = 2\sqrt{29}$	M1, A1
		(3)
(b)	States or implies that grad AB = $\frac{2}{5}$	B1
	Uses perpendicular gradient rule $\Rightarrow$ grad $l_2 = -\frac{5}{2}$	M1
	$y - "6" = "-\frac{5}{2}"(x-10) \Rightarrow 5x + 2y - 62 = 0$	dM1, A1
		(4)
(c)	C is $\left(\frac{62}{5}, 0\right)$	B1 ft
	$(BC^2) = \left(\frac{62}{5} - 10\right)^2 + ("6"-0)^2 \Rightarrow BC = \frac{6\sqrt{29}}{5} = (6.46)$	
	Area $ABCD$ = 69.6	M1, A1
		(3)
		(10 marks)

(a)

B1: States or implies that B is $(10, 6)$. May be awarded for sight of $x = 10,\ y = 6$ without reference to "B"

It is implied via sight of $(10-0)^2 + (6-p)^2$. Also be aware the answer may be on the Figure

M1: Attempts AB or $(AB)^2$ between $(0, 2)$ and their $(10, "6")$ **This must be in part (a) and not in part (c).**

Award for $\sqrt{(10-0)^2 + ("6"-2)^2}$ or $(10-0)^2 + ("6"-2)^2$ It is implied by $AB = \sqrt{116}$ or $AB^2 = 116$

There are other methods including $BH - AH = \sqrt{15^2 + 6^2} - \sqrt{5^2 + 2^2}$ where H is the x intercept of l_1

A1: $AB = 2\sqrt{29}$. This isn't a non-calculator question so minimal working is condoned/allowed.

(b)

B1: States or implies that grad l_1 = $\frac{2}{5}$ o.e such as 0.4, $\frac{6}{15}$ It is implied by grad $l_2 = -\frac{5}{2}$

It cannot be awarded for just writing the equation of l_1 as $y = \frac{2}{5}x + \ldots$.

Candidates must know which part of the equation is the gradient

M1: Uses perpendicular gradient rule $\Rightarrow$ grad $l_2 = -\frac{1}{\text{grad } l_1}$

dM1: Full attempt at equation of line l_2 $y - "6" = "-\frac{5}{2}"(x-10)$

For this to be awarded

- the gradient of l_2 must be either $-\frac{1}{\text{their grad } l_1}$ or if that was not attempted $-\frac{5}{2}$ or $-\frac{1}{0.4}$
- the point used must be $x = 10$ and their $y = 6$ (see part (a) for their B)

If the form $y = mx + k$ is used then the method must proceed as far as k = ...

A1: $5x + 2y - 62 = 0$ or any integer multiple of this

....

Special case in (b)

As the equation for l_1 is given as $5y = 2x + 10$ some candidates will assume the gradient is 2.

These candidates should get the equation of l_2 as $y - "6" = -\frac{1}{2}(x - 10)$

Such a candidate can score B0, M1, dM1, A0

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(c)

B1ft: States or implies that *C* is $\left(\frac{62}{5}, 0\right)$ o.e but follow through on their $-\frac{c}{a}$ from their $ax + by + c = 0$

M1: Attempts full method to find area *ABCD*. E.g $AB \times BC$ with a full attempt at *BC* using a correct method via Pythagoras' theorem. Follow through on their length *AB* from part (a)
If *A*, *B* and *C* are correct score for $AB = \text{awrt } 10.77, BC = 6.46$ followed by 10.77×6.46

A1: Exact answer is 69.6 or $\frac{348}{5}$ o.e. but accept awrt 69.6

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Other methods will be seen here, so look carefully at what is attempted.
E.g. The Shoelace Method
For example using *ABCD* which requires all 4 coordinates

$$\frac{1}{2} \times \begin{vmatrix} 0 & 10 & 12.4 & 2.4 & 0 \\ 2 & 6 & 0 & -4 & 2 \end{vmatrix} = \frac{1}{2} \times \left| 0 \times 6 + 10 \times 0 + 12.4 \times -4 + 2.4 \times 2 - 10 \times 2 - 12.4 \times 6 - 2.4 \times 0 - 0 \times -4 \right|$$

$$= \frac{1}{2} \times 139.2 = 69.6$$

B1ft: For *C* as in the main method

M1: Correct method for coordinate *D* (usually using vectors with $\overrightarrow{AD} = \overrightarrow{BC}$) followed by a full application of the method with correct signs. Score for an unsimplified expression following through on their *C*

A1: AWRT 69.6

Or more easily by doubling area triangle *ABC* which does not use coordinate *D*

$$2 \times \frac{1}{2} \times \begin{vmatrix} 0 & 10 & 12.4 & 0 \\ 2 & 6 & 0 & 2 \end{vmatrix} = \left| 0 \times 6 + 10 \times 0 + 12.4 \times 2 - 10 \times 2 - 12.4 \times 6 - 0 \times 0 \right|$$

$$= 69.6$$

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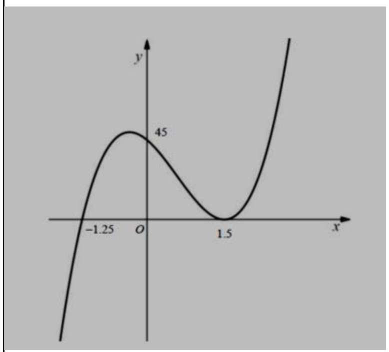
Question Number	Scheme	Marks
8 (a)	$f'(x) = 2(x-3)(3x+2) = 6x^2 - 14x - 12$	B1
	$f(x) = 2x^3 - 7x^2 - 12x + k$	M1, A1
	Uses $P(4, 13) \Rightarrow 13 = 2\times 64 - 7\times 16 - 12\times 4 + k \Rightarrow k = ...$	M1
	$(f(x)) = 2x^3 - 7x^2 - 12x + 45$	A1
		(5)
(b)	$2x^3 - 7x^2 - 12x + 45 \equiv (x^2 - 6x + 9)(px + q) \Rightarrow p = "2"$ or $q = \frac{"45"}{9}$	B1 ft
	States either $2x^3 - 7x^2 - 12x + 45 \equiv (x-3)^2(2x+5)$ or $p = 2$ & $q = 5$ following a fully correct (a) or correct but incomplete part (a) (See notes)	B1
		(2)
(c)	 + ve cubic curve	B1
	"Correct" y intercept for their part (b) equation. So $9q$ or "45"	B1ft
	Turning point at (1.5, 0)	B1
	x intercept at (−1.25, 0)	B1
		(4)
		(11 marks)

Mark parts (a) and (b) together but f(x) must be obtained via integration AND the use of (4, 13)

(a)

B1: $2(x-3)(3x+2) = 6x^2 - 14x - 12$ which may be left unsimplified BUT the brackets must be expanded

M1: Attempts to multiply out $2(x-3)(3x+2)$ and integrate each term.

Look for an expanded form which would simplify to $ax^2 + bx + c$ and **then each term** being integrated with each of the powers being correctly dealt with. So $ax^2 \to ...x^3, bx \to ...x^2$ and $+c \to cx$

There is no requirement to have a constant of integration for this mark

A1: $f(x) = 2x^3 - 7x^2 - 12x + k$ including the $+k$. (Any constant is acceptable)

The expression must now be in simplest form

M1: Uses P(4, 13) and finds a value for k. It is dependent upon some 'correct' integration.

So, look for substituting $x = 4, y = 13$ into their $y = f(x)$ and finding a value for k.

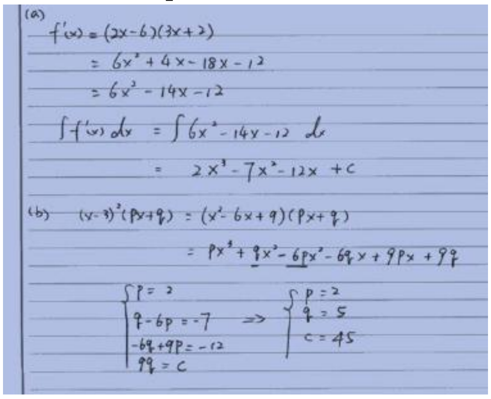
A1: $f(x) = 2x^3 - 7x^2 - 12x + 45$. There is no requirement to see the f (x)

(b)

B1ft: Achieves either value following through on their $2x^3 - 7x^2 - 12x + 45$.
So score for either of

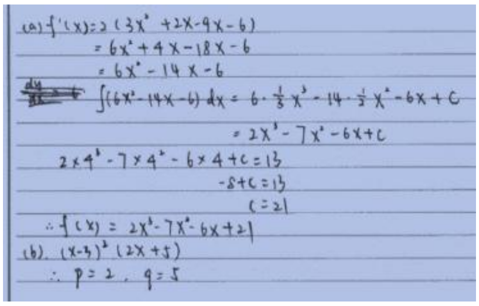
- p = 2 or the coefficient of their x^3 term
- q = 5 or their constant term divided by 9

B1: $(x-3)^2(2x+5)$ OR p = 2 or q = 5.
Must follow a fully correct part (a) = (1,1,1,1,1) or a correct but incomplete part (a) = (1,1,1,0,0)
Cannot be scored from multiple answers or implied from an attempt at division
Note that finding the two correct values is considered sufficient as long as $\mathrm{f}(x) = 2x^3 - 7x^2 - 12x + 45$
Useful examples



E.g I
(a) 1, 1, 1, 0, 0
No use of (4, 13) at all in the question so the final two marks cannot be awarded. (Note that they do find c = 45 in part (b) from expanding $(x-3)^2(2x+5)$ but this does not score those marks.)
(b) 1, 1
Correct values in (b) following a correct but incomplete part (a).
Note that $2x^3 - 7x^2 - 12x + c = (x-3)^2(2x+5)$ when c = 45.

Note that they could go on to score all marks in part (c)



E.g II
(a) 0, 1, 0, 1, 0
Incorrect expansion in (a) but uses (4, 13) to find c
(b)1, 0
Not a fully correct (a)
$2x^3 - 7x^2 - 6x - 21 \neq (x-3)^2(2x+5)$
Note that they could go on to score all marks in part (c) using the answer to part (b)

(c)

B1: For a positive cubic shaped curve with, from left to right, a maximum and then a minimum.
Do not be concerned by the position of the curve or the turning points. Be generous on graphs that appear linear in places but penalise three straight lines. Use review if unclear.

B1ft: For a correct y intercept for their $y = \mathrm{f}(2x)$. Don't be concerned if this is their maximum turning point.
FT on their answer to (a) for their '$2x^3 - 7x^2 - 12x + \underline{\underline{45}}$' provided that the constant term is not zero or their answer to (b) for their 9q provided $q \neq 0$

B1: Turning point tangential to the x -axis at 1.5. Allow for maximum or minimum point
Do not allow if marked (0, 1.5)

B1: x intercept at -1.25. The graph must pass through (not stop at) the axis here. Allow if there are other intercepts. Do not allow if marked (0, - 1.25).

Question Number	Scheme	Marks
9. (a)	(i) 3	B1
	(ii) 101	B1
	(iii) 2	B1
		(3)
(b)	$a=\frac{2\pi}{3}$ o.e.	B1
		(1)
(c)	Attempts x coordinate of Q: $x=\text{"}\frac{2\pi}{3}\text{"}+\pi=\frac{5\pi}{3}$	M1
	Attempts both y coordinates $P\left(\frac{2\pi}{3},\frac{\pi}{3}\right)$ $Q\left(\frac{5\pi}{3},-\frac{2\pi}{3}\right)$	dM1
	Attempts mid-point $PQ=\left(\frac{1}{2}\left(\frac{2\pi}{3}+\frac{5\pi}{3}\right),\frac{1}{2}\left(\frac{\pi}{3}-\frac{2\pi}{3}\right)\right)$	ddM1
	$=\left(\frac{7\pi}{6},-\frac{\pi}{6}\right)$	A1
		(4)
		(8 marks)

The answers to parts (a) and (b) may very well appear in the question. Be aware of this when marking.

(a)(i) B1: 3

(a)(ii) B1: 101

(a)(iii) B1: 2

(b)

B1: $a=\frac{2\pi}{3}$ o.e. so accept $a=\frac{4\pi}{6}$. Condone $x\leftrightarrow a$

Do not accept/condone $120°$

(c)

M1: For

- either the correct x coordinate of Q which can be scored for $\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)$
- or an attempt at the x coordinate of Q by adding on π to the x coordinate of P

The following marks are dependent upon this, so if (c) starts M0, it is automatically dM0, ddM0, A0.

dM1: Attempts coordinates for **BOTH** P and Q using the equation $y=\pi-x$

It is dependent upon the previous method mark.

ddM1: Correct attempt at mid-point.

There must be an attempt to add the pairs of coordinates before halving the sum.

It is dependent upon BOTH the previous method marks **and the value of *a* lying in the region** $\frac{\pi}{2}<a<\pi$ **.** Look for $\left(\frac{a+(a+\pi)}{2},\frac{(\pi-a)+(\pi-(a+\pi))}{2}\right)$

A1: $\left(\frac{7\pi}{6},-\frac{\pi}{6}\right)$ but may be written as $x=\ldots, y=\ldots$

Question Number	Scheme	Marks
10	$y=\frac{2}{3}x^3-25x-\frac{56}{x}+\frac{194}{3}$	
(a)	$\frac{dy}{dx}=2x^2-25+\frac{56}{x^2}$	M1, A1
	Finds $\left.\frac{dy}{dx}\right\|_{x=2}=2\times4-25+\frac{56}{4}=-3$	dM1
	Equation of tangent $y+8="-3"(x-2)\Rightarrow y=-3x-2$ *	ddM1, A1*
		(5)
(b)	Sets $2x^2-25+\frac{56}{x^2}=-3$	M1
	$x^4-11x^2+28=0$	dM1, A1
	$(x^2-4)(x^2-7)=0\Rightarrow x^2=..$	ddM1
	$x=\sqrt{7}$ only	A1
		(5)
		(10 marks)

This is a non-calculator question so you must see evidence for all calculations/marks

(a)

M1: Correctly differentiates two of the four terms. The indices must be processed
So, look for two of $\frac{2}{3}x^3\rightarrow\frac{2}{3}\times3x^2$, $-25x\rightarrow-25$, $-\frac{56}{x}\rightarrow+\frac{56}{x^2}$, $+\frac{194}{3}\rightarrow0$

A1: Correct differentiation. Allow un-simplified

dM1: Substitutes $x=2$ into their $\frac{dy}{dx}$ (to find the gradient of the tangent).
Allow values to be embedded following the M mark E.g. $2\times2^2-25+\frac{56}{4}$
It is dependent upon the previous M mark, so there must be some correct differentiation.
If there is no working for this, it is implied by a correct value for their $\frac{dy}{dx}$

ddM1: Dependent upon both previous M's.
For a correct method of finding the equation of the tangent at (2, −8).
Look for $y+8=\left.\frac{dy}{dx}\right|_{x=2}(x-2)$
If the form $y=mx+c$ is used it must proceed as far as c = ...

A1*: CSO: $y=-3x-2$ following the award of previous M's and A.
This is a given answer so all aspects must be correct.

(b)

M1: Sets their $\frac{dy}{dx}=-3$ **to form an equation in *x***

At least one term must have been differentiated correctly for this to be awarded

dM1: Multiplies by x^2 to form a 3-term quadratic equation in x^2. Terms need not be on the same side of the = sign. Allow equivalents, so an equation in *t* where $t=x^2$

It is dependent upon a 'correct' starting point of $-3=px^2+q+\frac{r}{x^2}$

A1: Correct 3 TQ = 0 in x^2. The '= 0' may be implied by subsequent work. Allow equivalents

ddM1: Solves via an appropriate method to find a value for x^2 other than 4. Dependent upon the previous dM1.

Allow factorisation (usual way of applying this, i.e first and last terms) and formula to find x^2

Examples of allowable work:

For an equation of the type $2x^4-22x^2+56=0$

E.g. 1: Via factorisation look for $\left(2x^4-8\right)\left(x^2-7\right)=0\Rightarrow x^2=...$

E.g. 2: Via the formula condone $x^2=\frac{22\pm\sqrt{(-22)^2-4\times2\times56}}{4}$ (a fully correct numerical version of the quadratic formula) leading to 7 and 4.

If there is an error E.g. $x^2=\frac{22\pm\sqrt{-22^2-4\times2\times56}}{4}$ it can only be awarded if recovered to $x^2=\frac{22\pm\sqrt{36}}{4}$ or $x^2=\frac{22\pm6}{4}$ before the roots 7 and 4 are given

E.g. 3: $x^4-11x^2+28=0\Rightarrow\left(x^2-7\right)\left(x^2-4\right)=0\Rightarrow x=\pm\sqrt{7}$ as the $x^2=$ can be implied

E.g. 4: $x^4-11x^2+28=0$ Let $t=x^2$, $t^2-11t+28=0\Rightarrow(t-7)(t-4)=0\Rightarrow t=7,4$

Examples of work that will NOT score marks:

E.g.1 $2x^4-22x^2+56=0\Rightarrow x^2=4,7$ (No method seen)

E.g.2 $x^4-11x^2+28=0$ Let $t=x^2$, $t^2-11t+28=0\Rightarrow t=7$ (No factorisation)

E.g.3 $x^4-11x^2+28=0$, $x^2=\frac{11\pm\sqrt{(-11)^2+4\times1\times28}}{2}\Rightarrow x^2=7$ (Incorrect formula)

E.g 4 Using num-solv on TI-36XPro, $x^4-11x^2+28=0$ followed by $x=\sqrt{7}$

A1: $x=\sqrt{7}$ ONLY following the previous marks.